Polynomials and Nonlinear Functions

What You’ll Learn

- **Lesson 13-1** Identify and classify polynomials.
- **Lessons 13-2 through 13-4** Add, subtract, and multiply polynomials.
- **Lesson 13-5** Determine whether functions are linear or nonlinear.
- **Lesson 13-6** Explore different representations of quadratic and cubic functions.

Why It’s Important

You have studied situations that can be modeled by linear functions. Many real-life situations, however, are not linear. These can be modeled using nonlinear functions. **You will use a nonlinear function in Lesson 13-6 to determine how far a skydiver falls in 4.5 seconds.**

Key Vocabulary

- polynomial (p. 669)
- degree (p. 670)
- nonlinear function (p. 687)
- quadratic function (p. 688)
- cubic function (p. 688)
Prerequisite Skills  To be successful in this chapter, you’ll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 13.

For Lesson 13-1  
Determine the number of monomials in each expression.  
(For review, see Lesson 4-1.)

1. \(2x^3\)  
2. \(a + 4\)  
3. \(8s - 5t\)  
4. \(x^2 + 3x - 1\)  
5. \(\frac{1}{t}\)  
6. \(9x^3 + 6x^2 + 8x - 7\)

For Lesson 13-4  
Use the Distributive Property to write each expression as an equivalent algebraic expression.  
(For review, see Lesson 3-1.)

7. \(5(a + 4)\)  
8. \(2(3y - 8)\)  
9. \(-4(1 + 8n)\)  
10. \(6(x + 2y)\)  
11. \((9b - 9c)3\)  
12. \(5(q - 2r + 3s)\)

For Lesson 13-5  
Determine whether each equation is linear.  
(For review, see Lesson 8-2.)

13. \(y = x - 2\)  
14. \(y = x^2\)  
15. \(y = -\frac{1}{2}x\)

Make this Foldable to help you organize information about polynomials and nonlinear functions. Begin with a sheet of 11" × 17" paper.

Step 1  Fold  
Fold the short sides toward the middle.

Step 2  Fold Again  
Fold the top to the bottom.

Step 3  Cut  
Open. Cut along the second fold to make four tabs.

Step 4  Label  
Label each of the tabs as shown.

Reading and Writing  As you read and study the chapter, write examples of each concept under each tab.
Prefixes and Polynomials

You can determine the meaning of many words used in mathematics if you know what the prefixes mean. In Lesson 4-1, you learned that the prefix \textit{mono} means one and that a monomial is an algebraic expression with one term.

<table>
<thead>
<tr>
<th>Monomials</th>
<th>Not Monomials</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>( x + y )</td>
</tr>
<tr>
<td>( 2x )</td>
<td>( 8n^2 - n + 1 )</td>
</tr>
<tr>
<td>( y^3 )</td>
<td>( a^3 + 4a^2 + a - 6 )</td>
</tr>
</tbody>
</table>

The words in the table below are used in mathematics and in everyday life. They contain the prefixes \textit{bi}, \textit{tri}, and \textit{poly}.

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>\textit{bi}</td>
<td>• bisect – to divide into two congruent parts&lt;br&gt;• biannual – occurring twice a year&lt;br&gt;• bicycle – a vehicle with two wheels</td>
</tr>
<tr>
<td>\textit{tri}</td>
<td>• triangle – a figure with three sides&lt;br&gt;• triathlon – an athletic contest with three phases&lt;br&gt;• trilogy – a series of three related literary works, such as films or books</td>
</tr>
<tr>
<td>\textit{poly}</td>
<td>• polyhedron – a solid with many flat surfaces&lt;br&gt;• polychrome – having many colors&lt;br&gt;• polygon – a figure with many sides</td>
</tr>
</tbody>
</table>

\textit{Reading to Learn}

1. How are the words in each group of the table related?
2. What do the prefixes \textit{bi}, \textit{tri}, and \textit{poly} mean?
3. Write the definition of \textit{binomial}, \textit{trinomial}, and \textit{polynomial}.
4. Give an example of a binomial, a trinomial, and a polynomial.
5. \textbf{RESEARCH} Use the Internet or a dictionary to make a list of other words that have the prefixes \textit{bi}, \textit{tri}, and \textit{poly}. Give the definition of each word.
Polynomials

What You’ll Learn

• Identify and classify polynomials.
• Find the degree of a polynomial.

Vocabulary

• polynomial
• binomial
• trinomial
• degree

How are polynomials used to approximate real-world data?

Heat index is a way to describe how hot it feels outside with the temperature and humidity combined. Some examples are shown below.

To calculate heat index, meteorologists use an expression similar to the one below. In this expression, \(x\) is the percent humidity, and \(y\) is the temperature.

\[-42 + 2x + 10y - 0.2xy - 0.007x^2 - 0.05y^2 + 0.001x^2y + 0.009xy^2 - 0.000002x^2y^2\]

a. How many terms are in the expression for the heat index?
b. What separates the terms of the expression?

CLASSIFY POLYNOMIALS

Recall that a monomial is a number, a variable, or a product of numbers and/or variables. An algebraic expression that contains one or more monomials is called a polynomial. In a polynomial, there are no terms with variables in the denominator and no terms with variables under a radical sign.

A polynomial with two terms is called a binomial, and a polynomial with three terms is called a trinomial.

The terms in a binomial or a trinomial may be added or subtracted.

Example 1 Classify Polynomials

Determine whether each expression is a polynomial. If it is, classify it as a monomial, binomial, or trinomial.

a. \(2x^3 + 5x + 7\)
   
   This is a polynomial because it is the sum of three monomials. There are three terms, so it is a trinomial.

b. \(t - \frac{1}{t^2}\)
   
   The expression is not a polynomial because \(\frac{1}{t^2}\) has a variable in the denominator.

Concept Check

Is \(0.5x + 10\) a polynomial? Explain.
The degree of a monomial is the sum of the exponents of its variables. The degree of a nonzero constant such as 6 or 10 is 0. The constant 0 has no degree.

**Example 2  Degree of a Monomial**

Find the degree of each monomial.

a. \(5a\)

   The variable \(a\) has degree 1, so the degree of \(5a\) is 1.

b. \(-3x^2y\)

   \(x^2\) has degree 2 and \(y\) has degree 1. The degree of \(-3x^2y\) is \(2 + 1 \text{ or } 3\).

A polynomial also has a degree. The degree of a polynomial is the same as that of the term with the greatest degree.

**Example 3  Degree of a Polynomial**

Find the degree of each polynomial.

a. \(x^2 + 3x - 2\)

   The greatest degree is 2. So the degree of \(x^2 + 3x - 2\) is 2.

b. \(a^2 + ab^2 + b^4\)

   The greatest degree is 4. So the degree of \(a^2 + ab^2 + b^4\) is 4.

**Example 4  Degree of a Real-World Polynomial**

ECOLOGY  In the early 1900s, the deer population of the Kaibab Plateau in Arizona was affected by hunters and by the food supply. The population from 1905 to 1930 can be approximated by the polynomial \(-0.13x^5 + 3.13x^4 + 4000\), where \(x\) is the number of years since 1900. Find the degree of the polynomial.

\[
-0.13x^5 + 3.13x^4 + 4000
\]

So, \(-0.13x^5 + 3.13x^4 + 4000\) has degree 5.

**Concept Check**  Find the degree of the polynomial at the beginning of the lesson.
3. **FIND THE ERROR** Carlos and Tanisha are finding the degree of $5x + y^2$.

Carlos
- $5x$ has degree 1.
- $y^2$ has degree 2.
- $5x + y^2$ has degree $1 + 2$ or 3.

Tanisha
- $5x$ has degree 1.
- $y^2$ has degree 2.
- $5x + y^2$ has degree 2.

Who is correct? Explain your reasoning.

**Guided Practice**

Determine whether each expression is a polynomial. If it is, classify it as a *monomial, binomial, or trinomial*.

4. $-7$
5. $\dfrac{d}{2}$
6. $\dfrac{1}{x} - x$
7. $a^5 + a^3$
8. $y^2 - 4$
9. $x^2 + xy^2 - y^2$

Find the degree of each polynomial.

10. $4b^2$
11. $121$
12. $8x^3y^2$
13. $3x + 5$
14. $r^3 + 7r$
15. $d^2 + c^4$

**Application**

**GEOMETRY** For Exercises 16 and 17, refer to the square at the right with a side length of $x$ units.

16. Write a polynomial expression for the area of the small blue rectangle.
17. What is the degree of the polynomial you wrote in Exercise 16?

**Practice and Apply**

Determine whether each expression is a polynomial. If it is, classify it as a *monomial, binomial, or trinomial*.

18. $16$
19. $x^2 - 7x$
20. $11a^2 + 4$
21. $-\dfrac{1}{3}a^2$
22. $\sqrt{15c}$
23. $8 - \dfrac{2}{k}$
24. $r^4 + r^2s^2$
25. $12 - n + n^4$
26. $ab^2 + 3a - b^2$
27. $\sqrt{y} + y$
28. $\dfrac{ab}{c} - c$
29. $x^2 - \dfrac{1}{2}x + \dfrac{1}{3}$

Find the degree of each polynomial.

30. $3$
31. $56$
32. $ab$
33. $12c^3$
34. $xyz^2$
35. $9s^4t$
36. $2 - 8n$
37. $g^5 + 5h$
38. $x^2 + 3x + 2$
39. $4y^3 + 6y^2 - 5y - 1$
40. $d^2 + cd^2$
41. $x^3 - x^2y^3 + 8$

Tell whether each statement is *always, sometimes, or never* true. Explain.

42. A trinomial has a degree of 3. 43. An integer is a monomial.

44. **MEDICINE** Doctors can study a patient’s heart by injecting dye in a vein near the heart. In a normal heart, the amount of dye in the bloodstream after $t$ seconds is given by $-0.006t^4 + 0.140t^3 - 0.53t^2 + 1.79t$. Find the degree of the polynomial.
LANDSCAPING  For Exercises 45 and 46, use the information below and the diagram at the right. Lee wants to plant flowers along the perimeter of his vegetable garden.

45. Write a polynomial that represents the perimeter of the garden in feet.

46. What is the degree of the polynomial?

47. RESEARCH  Suppose your grandparents deposited $100 in your savings account each year on your birthday. On your fifth birthday, there would have been approximately $100x^4 + 100x^3 + 100x^2 + 100x + 100$ dollars, where $x$ is the annual interest rate plus 1. Research the current interest rate at your family’s bank. Using that interest rate, how much money would you have on your next birthday?

48. CRITICAL THINKING  Find the degree of $ax^3 + bx^2 + c$.

49. WRITING IN MATH  Answer the question that was posed at the beginning of the lesson.

How are polynomials used to approximate real-world data?
Include the following in your answer:
• a description of how the value of heat index is found, and
• an explanation of why a linear equation cannot be used to approximate the heat index data.

50. Choose the expression that is not a binomial.
   A) $x^2 - 1$  B) $a + b$  C) $m^3 + n^3$  D) $7x + 2x$

51. State the degree of $4x^3 + xy - y^2$.
   A) 1  B) 2  C) 3  D) 4

Mixed Review  A number cube is rolled. Determine whether each event is mutually exclusive or inclusive. Then find the probability.  (Lesson 12-9)

52. $P$(odd or greater than 3)  53. $P$(5 or even)

54. A number cube is rolled. Find the odds that the number is greater than 2.  (Lesson 12-8)

Find the volume of each solid. If necessary, round to the nearest tenth.  (Lesson 11-3)

55. 56.  11.3 m  4 m  7 in.  5 m  6 in.

Getting Ready for the Next Lesson  PREREQUISITE SKILL  Rewrite each expression using parentheses so that the terms having variables of the same power are grouped together. (To review properties of addition, see Lesson 1-4.)

57. $(x + 4) + 2x$  58. $3x^2 - 1 + x^2$  59. $(6n + 2) + (3n + 5)$

60. $(a + 2b) + (3a + b)$  61. $(s + t) + (5s - 3t)$  62. $(x^2 + 4x) + (7x^2 - 3x)$
Investigating Slope-Intercept Form

Algebra Activity

Modeling Polynomials with Algebra Tiles

A Follow-Up of Lesson 13-1

In a set of algebra tiles, \( \square \) represents the integer 1, \( x \) represents the variable \( x \), and \( x^2 \) represents \( x^2 \). Red tiles are used to represent \(-1\), \(-x\), and \(-x^2\).

You can use these tiles to model monomials.

You can also use algebra tiles to model polynomials. The polynomial \( 2x^2 - 3x + 4 \) is modeled below.

Model and Analyze

Use algebra tiles to model each polynomial.

1. \(-3x^2\)
2. \(5x + 3\)
3. \(4x^2 - x\)
4. \(2x^2 + 2x - 3\)

5. Explain how you can tell whether an expression is a monomial, binomial, or trinomial by looking at the algebra tiles.

6. Name the polynomial modeled below.

7. Explain how you would find the degree of a polynomial using algebra tiles.
**What You’ll Learn**

- Add polynomials.

**How can you use algebra tiles to add polynomials?**

Consider the polynomials $2x^2 - 3x + 4$ and $-x^2 + x - 2$ modeled below.

Follow these steps to add the polynomials.

**Step 1** Combine the tiles that have the same shape.

**Step 2** When a positive tile is paired with a negative tile that is the same shape, the result is called a zero pair. Remove any zero pairs.

a. Write the polynomial for the tiles that remain.

b. Find the sum of $x^2 + 4x + 2$ and $7x^2 - 2x + 3$ by using algebra tiles.

c. Compare and contrast finding the sums of polynomials with finding the sum of integers.

### ADD POLYNOMIALS

Monomials that contain the same variables to the same power are like terms. Terms that differ only by their coefficient are called like terms.

**Like Terms**
- $2x$ and $7x$
- $-x^2y$ and $5x^2y$

**Unlike Terms**
- $-6a$ and $7b$
- $4ab^2$ and $4a^2b$

You can add polynomials by combining like terms.

**Example 1** Add Polynomials

Find each sum.

a. $(3x + 5) + (2x + 1)$

**Method 1** Add vertically.

\[
\begin{align*}
3x + 5 \\
+ (2x + 1) \\
5x + 6
\end{align*}
\]

Align like terms. Add.

The sum is $5x + 6$.

**Method 2** Add horizontally.

\[
(3x + 5) + (2x + 1) = (3x + 2x) + (5 + 1) = 5x + 6
\]
b. \((2x^2 + x - 7) + (x^2 + 3x + 5)\)

**Method 1**

\[
\begin{align*}
2x^2 & \quad + \quad x - 7 \\
(+) & \quad x^2 + 3x + 5 \\
\underline{3x^2 + 4x - 2} & \quad \text{Align like terms.}
\end{align*}
\]

**Method 2**

\[
\begin{align*}
(2x^2 + x - 7) + (x^2 + 3x + 5) & \quad \text{Write the expression.} \\
= (2x^2 + x^2) + (x + 3x) + (-7 + 5) & \quad \text{Group like terms.} \\
= 3x^2 + 4x - 2 & \quad \text{Simplify.}
\end{align*}
\]

The sum is \(3x^2 + 4x - 2\).

c. \((9c^2 + 4c) + (-6c + 8)\)

\[
\begin{align*}
(9c^2 + 4c) + (-6c + 8) & \quad \text{Write the expression.} \\
= 9c^2 + (4c - 6c) + 8 & \quad \text{Group like terms.} \\
= 9c^2 - 2c + 8 & \quad \text{Simplify.}
\end{align*}
\]

The sum is \(9c^2 - 2c + 8\).

d. \((x^2 + xy + 2y^2) + (6x^2 - y^2)\)

\[
\begin{align*}
& \quad x^2 + xy + 2y^2 \\
(+) & \quad 6x^2 - y^2 \\
7x^2 + xy & \quad \text{Leave a space because there is no other term like } xy.
\end{align*}
\]

The sum is \(7x^2 + xy + y^2\).

**Concept Check**

Name the like terms in \(b^2 + 5b - ab + 9b^2\).

Polynomials are often used to represent measures of geometric figures.

**Example 2**

**Use Polynomials to Solve a Problem**

**GEOMETRY**

The lengths of the sides of golden rectangles are in the ratio \(1:1.62\). So, the length of a golden rectangle is approximately 1.62 times greater than the width.

a. Find a formula for the perimeter of a golden rectangle.

\[
P = 2\ell + 2w \\
P = 2(1.62x) + 2x \quad \text{Formula for the perimeter of a rectangle} \\
P = 3.24x + 2x \quad \text{Replace } \ell \text{ with } 1.62x \text{ and } w \text{ with } x. \\
P = 5.24x \quad \text{Simplify.}
\]

A formula for the perimeter of a golden rectangle is \(P = 5.24x\), where \(x\) is the measure of the width.

b. Find the length and the perimeter of a golden rectangle if its width is 8.3 centimeters.

\[
\begin{align*}
\text{length} & \quad = 1.62x \\
& \quad = 1.62(8.3) \text{ or } 13.446 \quad \text{Length of a golden rectangle} \\
\text{perimeter} & \quad = 5.24x \\
& \quad = 5.24(8.3) \text{ or } 43.492 \quad \text{Perimeter of a golden rectangle}
\end{align*}
\]

The length of the golden rectangle is 13.446 centimeters, and the perimeter is 43.492 centimeters.
1. Name the like terms in \((x^2 + 5x + 2) + (2x^2 - 4x + 7)\).

2. OPEN ENDED Write two binomials that share only one pair of like terms.

3. FIND THE ERROR Hai says that \(7xyz\) and \(2zyx\) are like terms. Devin says they are not. Who is correct? Explain your reasoning.

Guided Practice
Find each sum.
4. \(4x + 5 + (-x - 3)\)
5. \(3a^2 - 9a + 6 + (4a^2 - 2)\)
6. \((x + 3) + (2x + 5)\)
7. \((13x - 7y) + 3y\)
8. \((2x^2 + 5x) + (9 - 7y)\)
9. \((3x^2 - 2x + 1) + (x^2 + 5x - 3)\)

Application
10. GEOMETRY Find the perimeter of the figure at the right.

Practice and Apply
Homework Help
For Exercises See Examples
11–22 1
23–27 2
Extra Practice
See page 756.

Find each sum.
11. \(-5x + 4 + 8x - 1\)
12. \(7b - 5 - 9b + 8\)
13. \(10x^2 + 5xy + 7y^2 + x^2 - 3y^2\)
14. \(4a^3 + a^2 + 8a - 8 + 2a^2 + 6\)
15. \((3x + 9) + (x + 5)\)
16. \((4x + 3) + (x - 1)\)
17. \((6y - 5r) + (2y + 7r)\)
18. \((8m - 2n) + (3m + n)\)
19. \((x^2 + y) + (4x^2 + xy)\)
20. \((3a^2 + b^2) + (3a + b^2)\)
21. \((5x^2 + 6x + 4) + (2x^2 + 3x + 1)\)
22. \((-2x^2 + x - 5) + (x^2 - 3x + 2)\)

Find each sum. Then evaluate if \(a = -3\), \(b = 4\), and \(c = 2\).
23. \((3a + 5b) + (2a - 9b)\)
24. \((a^2 + 7b^2) + (5 - 3b^2) + (2a^2 - 7)\)
25. \((3a + 5b - 4c) + (2a - 3b + 7c) + (-a + 4b - 2c)\)

GEOMETRY For Exercises 26–28, refer to the triangle.
26. Find the sum of the measures of the angles.
27. The sum of the measures of the angles in any triangle is \(180^\circ\). Find the value of \(x\).
28. Find the measure of each angle.
FINANCE  For Exercises 29–31, refer to the information below.
Jason and Will both work at the same supermarket and are paid the same hourly rate. At the end of the week, Jason’s paycheck showed that he worked 23 hours and had $12 deducted for taxes. Will worked 19 hours during the same week and had $10 deducted for taxes. Let \( x \) represent the hourly pay.

29. Write a polynomial expression to represent Jason’s pay for the week.
30. Write a polynomial expression to represent Will’s pay for the week.
31. Write a polynomial expression to represent the total weekly pay for Jason and Will.

32. CRITICAL THINKING  In the figure at the right, \( x^2 \) is the area of the larger square, and \( y^2 \) is the area of each of the two smaller squares. What is the perimeter of the whole rectangle? Explain.

33. WRITING IN MATH  Answer the question that was posed at the beginning of the lesson.

How can you use algebra tiles to add polynomials?
Include the following in your answer:
• a description of algebra tiles that represent like terms, and
• an explanation of how zero pairs are used in adding polynomials.

34. Choose the pair of terms that are not like terms.
   \[ A \quad 6cd, 12cd \quad B \quad \frac{x}{2}, 5x \quad C \quad a^2, b^2 \quad D \quad x^2y, yx^2 \]

35. What is the sum of \( 11x + 2y \) and \( x - 5y \)?
   \[ A \quad 10x - 3y \quad B \quad 12x - 3y \quad C \quad 12x + 3y \quad D \quad 12x - 5y \]

Maintain Your Skills

Mixed Review  Find the degree of each polynomial.  (Lesson 13-1)

36. \( a^3b \)  
37. \( 3x - 5y + x^2 \)  
38. \( c^2 - 7c^3y^4 \)

A card is drawn from a standard deck of 52 playing cards. Find each probability.  (Lesson 12-9)

39. \( P(2 \text{ or jack}) \)  
40. \( P(10 \text{ or red}) \)  
41. \( P(\text{ace or black } 7) \)

42. Determine whether the prisms are similar. Explain.  (Lesson 11-6)

Getting Ready for the Next Lesson  PREREQUISITE SKILL  Rewrite each expression as an addition expression by using the additive inverse.  (To review additive inverse, see Lesson 2-3.)

43. \( 15c - 26 \)  
44. \( x^2 - 7 \)  
45. \( 1 - 2x \)

46. \( 6b - 3a^2 \)  
47. \( (n + rt) - r^2 \)  
48. \( (s + t) - 2s \)

www.pre-alg.com/self_check_quiz/fcat
**What You’ll Learn**

- Subtract polynomials.

**How is subtracting polynomials similar to subtracting measurements?**

At the North Pole, buoy stations drift with the ice in the Arctic Ocean. The table shows the latitudes of two North Pole buoys in April, 2000.

<table>
<thead>
<tr>
<th>Station</th>
<th>Latitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>89° 35.4’N = 89 degrees 35.4 minutes</td>
</tr>
<tr>
<td>5</td>
<td>85° 27.3’N = 85 degrees 27.3 minutes</td>
</tr>
</tbody>
</table>

**Reading Math**

**Symbols**

The symbol "" in 68° 8’ 2” is read as seconds.

**Example 1**

**Subtract Polynomials**

Find each difference.

a. \((5x + 9) - (3x + 6)\)

\[
\begin{align*}
5x + 9 &\quad \text{Align like terms.} \\
- (3x + 6) &\quad \text{Align like terms.} \\
2x + 3 &\quad \text{Subtract.}
\end{align*}
\]

The difference is \(2x + 3\).

b. \((4a^2 + 7a + 4) - (3a^2 + 2)\)

\[
\begin{align*}
4a^2 + 7a + 4 &\quad \text{Align like terms.} \\
- (3a^2 + 2) &\quad \text{Align like terms.} \\
a^2 + 7a + 2 &\quad \text{Subtract.}
\end{align*}
\]

The difference is \(a^2 + 7a + 2\).
Recall that you can subtract a rational number by adding its additive inverse.

\[10 - 8 = 10 + (-8)\] The additive inverse of 8 is \(-8\).

You can also subtract a polynomial by adding its additive inverse. To find the additive inverse of a polynomial, multiply the entire polynomial by \(-1\).

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Multiply by (-1)</th>
<th>Additive Inverse</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t)</td>
<td>(-1(t))</td>
<td>(-t)</td>
</tr>
<tr>
<td>(x + 3)</td>
<td>(-1(x + 3))</td>
<td>(-x - 3)</td>
</tr>
<tr>
<td>(-a^2 + b^2 - c)</td>
<td>(-1(-a^2 + b^2 - c))</td>
<td>(a^2 - b^2 + c)</td>
</tr>
</tbody>
</table>

### Example 2 Subtract Using the Additive Inverse

Find each difference.

a. \((3x + 8) - (5x + 1)\)

The additive inverse of \(5x + 1\) is \((-1)(5x + 1)\) or \(-5x - 1\).

\[
(3x + 8) - (5x + 1) = (3x + 8) + (-5x - 1)
\]

To subtract \((5x + 1)\), add \((-5x - 1)\).

Group the like terms.

Simplify.

The difference is \(-2x + 7\).

b. \((4x^2 + y^2) - (-3xy + y^2)\)

The additive inverse of \(-3xy + y^2\) is \((-1)(-3xy + y^2)\) or \(3xy - y^2\).

Align the like terms and add the additive inverse.

\[
\frac{4x^2 + y^2}{(-)} - 3xy + y^2 \rightarrow \frac{4x^2 + y^2}{(+)} 3xy - y^2
\]

The difference is \(4x^2 + 3xy\).

### Concept Check

What is the additive inverse of \(a^2 + 9a - 1\)?

### Example 3 Subtract Polynomials to Solve a Problem

**SHIPPING** The cost for shipping a package that weighs \(x\) pounds from Dallas to Chicago is shown in the table at the right. How much more does the Atlas Service charge for shipping the package?

<table>
<thead>
<tr>
<th>Shipping Company</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlas Service</td>
<td>4x + 280</td>
</tr>
<tr>
<td>Bell Service</td>
<td>3x + 125</td>
</tr>
</tbody>
</table>

difference in cost = cost of Atlas Service - cost of Bell Service

\[
= (4x + 280) - (3x + 125)
\]

Substitution

\[
= (4x + 280) + (-3x - 125)
\]

Add additive inverse.

\[
= (4x - 3x) + (280 - 125)
\]

Group like terms.

Simplify.

The Atlas Service charges \(x + 155\) dollars more for shipping a package that weighs \(x\) pounds.
Check for Understanding

Concept Check

1. Describe how subtraction and addition of polynomials are related.

2. OPEN ENDED Write two polynomials whose difference is \( x^2 + 2x - 4 \).

Guided Practice

Find each difference.

3. \[ r^2 + 5r \\ (-) r^2 + r \]

4. \[ 3x^2 + 5x + 4 \\ (-) x^2 - 1 \]

5. \[ (9x + 5) - (4x + 3) \]

6. \[ (2x + 4) - (-x + 5) \]

7. \[ (3x^2 + x) - (8 - 2x) \]

8. \[ (6a^2 - 3a + 9) - (7a^2 + 5a - 1) \]

Application

9. GEOMETRY The perimeter of the isosceles trapezoid shown is \( 16x + 1 \) units. Find the length of the missing base of the trapezoid.

Practice and Apply

Find each difference.

10. \[ 8k + 9 \\ (-) k + 2 \]

11. \[ -n^2 + 1n \\ (-) n^2 - 5n \]

12. \[ 5a^2 + 9a - 12 \\ (-) -3a^2 + 5a - 7 \]

13. \[ 6y^2 - 5y + 3 \\ (-) 5y^2 + 2y - 7 \]

14. \[ 5x^2 - 4xy \\ (-) 3xy + 2y^2 \]

15. \[ 9w^2 + 7 \\ (-) -6w^2 + 2w - 3 \]

16. \[ (3x + 4) - (x + 2) \]

17. \[ (7x + 5) - (3x + 2) \]

18. \[ (2y + 5) - (y + 8) \]

19. \[ (3t - 2) - (5t - 4) \]

20. \[ (2x + 3y) - (x - y) \]

21. \[ (a^2 + 6b^2) - (-2a^2 + 4b^2) \]

22. \[ (x^2 + 6x) - (3x^2 + 7) \]

23. \[ (9n^2 - 8) - (n + 4) \]

24. \[ (6x^2 + 3x + 9) - (2x^2 + 8x + 1) \]

25. \[ (3x^2 - 5xy + 7y^2) - (x^2 - 3xy + 4y^2) \]

26. GEOMETRY Alyssa plans to trim a picture to fit into a frame. The area of the picture is \( 2x^2 + 11x + 12 \) square units, but the area inside the frame is only \( 2x^2 + 5x + 2 \) square units. How much of the picture will Alyssa have to trim so that it will fit into the frame?

27. TEMPERATURE The highest recorded temperature in North Carolina occurred in 1983. The lowest recorded temperature in North Carolina occurred two years later. The difference between these two record temperatures is \( 68^\circ F \) more than the sum of the temperatures. Write an equation to represent this situation. Then find the record low temperature in North Carolina.

28. CRITICAL THINKING Suppose \( A \) and \( B \) represent polynomials. If \( A + B = 3x^2 + 2x - 2 \) and \( A - B = -x^2 + 4x - 8 \), find \( A \) and \( B \).
29. **WRITING IN MATH**
   Answer the question that was posed at the beginning of the lesson.

   How is subtracting polynomials similar to subtracting measurements?
   Include the following in your answer:
   • a comparison between subtracting measurements with two parts and subtracting polynomials with two terms, and
   • an example of a subtraction problem involving measurements that have two parts, and an explanation of how to find the difference.

30. What is \((5x - 7) - (3x - 4)\)?
   A. 2x - 3  
   B. 2x + 3  
   C. 2x - 11  
   D. 2x + 11

31. Write the additive inverse of \(-4h^2 - hk - k^2\).
   A. \(4h^2 + hk + k^2\)  
   B. \(-4h^2 + hk + k^2\)  
   C. \(-4h^2 - hk - k^2\)  
   D. \(-4h^2 +hk - k^2\)

**Maintain Your Skills**

**Mixed Review**
Find each sum. *(Lesson 13-2)*

32. \((2x - 3) + (x - 1)\)  
33. \((11x + 2y) + (x - 5y)\)  
34. \((5x^2 - 7x + 9) + (3x^2 + 4x - 6)\)  
35. \((4t - t^2) + (8t + 2)\)

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL**
Simplify each expression.

*(To review multiplying monomials, see Lesson 4-2.)*

40. \(x(3x)\)  
41. \((2y)(4y)\)  
42. \((t^2)(6t)\)  
43. \((4m)(m^2)\)  
44. \((w^2)(-3w)\)  
45. \((2r^2)(5r^3)\)

**Practice Quiz 1**

Find the degree of each polynomial. *(Lesson 13-1)*

1. \(cd^3\)  
2. \(a - 4a^2\)  
3. \(x^2y + 7x^2 - 21\)

Find each sum or difference. *(Lessons 13-2 and 13-3)*

4. \((2x - 8) + (x - 7)\)  
5. \((4x + 5) - (2x + 3)\)  
6. \((5d^2 - 3) - (2d^2 - 7)\)  
7. \((3r + 6s) + (5r - 9s)\)  
8. \((x^2 + 4x + 2) + (7x^2 - 2x + 3)\)  
9. \((9x - 4y) - (12x - 9y)\)

10. **GEOMETRY**
    The perimeter of the triangle is \(8x + 3y\) centimeters.
    Find the length of the third side. *(Lessons 13-2 and 13-3)*
Modeling Multiplication

Recall that algebra tiles are named based on their area. The area of each tile is the product of the width and length.

These algebra tiles can be placed together to form a rectangle whose length and width each represent a polynomial. The area of the rectangle is the product of the polynomials.

Use algebra tiles to find $x(x + 2)$.

**Step 1** Make a rectangle with a width of $x$ and a length of $x + 2$. Use algebra tiles to mark off the dimensions on a product mat.

**Step 2** Using the marks as a guide, fill in the rectangle with algebra tiles.

**Step 3** The area of the rectangle is $x^2 + x + x$. In simplest form, the area is $x^2 + 2x$. Therefore, $x(x + 2) = x^2 + 2x$.

**Model and Analyze**

Use algebra tiles to determine whether each statement is true or false.

1. $x(x + 1) = x^2 + 1$
2. $x(2x + 3) = 2x^2 + 3x$
3. $(x + 2)2x = 2x^2 + 4x$
4. $2x(3x + 1) = 6x^2 + x$

Find each product using algebra tiles.

5. $x(x + 5)$
6. $(2x + 1)x$
7. $(2x + 4)2x$
8. $3x(2x + 1)$
9. There is a square garden plot that measures $x$ feet on a side.
   a. Suppose you double the length of the plot and increase the width by 3 feet. Write two expressions for the area of the new plot.
   b. If the original plot was 10 feet on a side, what is the area of the new plot?

**Extend the Activity**

10. Write a multiplication sentence that is represented by the model at the right.
Multiplying a Polynomial by a Monomial

What You’ll Learn

• Multiply a polynomial by a monomial.

How is the Distributive Property used to multiply a polynomial by a monomial?

The Grande Arche office building in Paris, France, looks like a hollowed-out prism, as shown in the photo at the right.

a. Write an expression that represents the area of the rectangular region outlined on the photo.

b. Recall that \(2(4 + 1) = 2(4) + 2(1)\) by the Distributive Property. Use this property to simplify the expression you wrote in part a.

c. The Grande Arche is approximately \(w\) feet deep. Explain how you can write a polynomial to represent the volume of the hollowed-out region of the building. Then write the polynomial.

MULTIPLY A POLYNOMIAL AND A MONOMIAL

You can model the multiplication of a polynomial and a monomial by using algebra tiles.

The model shows the product of \(2x\) and \(x + 3\). The rectangular arrangement contains \(2x^2\) tiles and 6 tiles. So, the product of \(2x\) and \(x + 3\) is \(2x^2 + 6x\). In general, the Distributive Property can be used to multiply a polynomial and a monomial.

Example 1 Products of a Monomial and a Polynomial

Find each product.

a. \(4(5x + 1)\)

\[
4(5x + 1) = 4(5x) + 4(1) \quad \text{Distributive Property}
\]

\[
= 20x + 4 \quad \text{Simplify.}
\]

b. \((2x - 6)(3x)\)

\[
(2x - 6)(3x) = 2x(3x) - 6(3x) \quad \text{Distributive Property}
\]

\[
= 6x^2 - 18x \quad \text{Simplify.}
\]
Sometimes problems can be solved by simplifying polynomial expressions.

**Concept Check**

What is the product of \( x^2 \) and \( x + 1 \)?

**Example 2**  
**Product of a Monomial and a Polynomial**

Find \( 3a(a^2 + 2ab - 4b^2) \).

\[
3a(a^2 + 2ab - 4b^2) = 3a(a^2) + 3a(2ab) - 3a(4b^2) \quad \text{Distributive Property}
\]

\[
= 3a^3 + 6a^2b - 12ab^2 \quad \text{Simplify.}
\]

**Example 3**  
**Use a Polynomial to Solve a Problem**

**POOLS**  
The world's largest swimming pool is the Orthlieb Pool in Casablanca, Morocco. It is 30 meters longer than 6 times its width. If the perimeter of the pool is 1110 meters, what are the dimensions of the pool?

**Explore**  
You know the perimeter of the pool. You want to find the dimensions of the pool.

**Plan**  
Let \( w \) represent the width of the pool. Then \( 6w + 30 \) represents the length. Write an equation.

\[
\text{Perimeter equals twice the sum of the length and width.}
\]

\[
P = 2(\ell + w)
\]

**Solve**  
\[
P = 2(\ell + w) \quad \text{Write the equation.}
\]

\[
1110 = 2(6w + 30 + w) \quad \text{Replace } P \text{ with } 1110 \text{ and } \ell \text{ with } 6w + 30.
\]

\[
1110 = 2(7w + 30) \quad \text{Combine like terms.}
\]

\[
1110 = 14w + 60 \quad \text{Distributive Property}
\]

\[
1050 = 14w \quad \text{Subtract 60 from each side.}
\]

\[
75 = w \quad \text{Divide each side by 14.}
\]

The width is 75 meters, and the length is \( 6w + 30 \) or 480 meters.

**Examine**  
Check the reasonableness of the results by estimating.

\[
P = 2(\ell + w) \quad \text{Formula for perimeter of a rectangle}
\]

\[
P \approx 2(500 + 80) \quad \text{Round 480 to 500 and 75 to 80.}
\]

\[
P \approx 2(580) \text{ or about } 1160
\]

Since 1160 is close to 1110, the answer is reasonable.

**Check for Understanding**

**Concept Check**

1. Determine whether the following statement is true or false.

   If you change the order in which you multiply a polynomial and a monomial, the product will be different.

   Explain your reasoning or give a counterexample.

2. Explain the steps you would take to find the product of \( x^3 - 7 \) and \( 4x \).

3. **OPEN ENDED**  
   Write a monomial and a polynomial, each having a degree no greater than 1. Then find their product.
Guided Practice

Find each product.
4. \((5y - 4)3\)
5. \(a(a + 4)\)
6. \(t(7t + 8)\)
7. \((3x - 7)4x\)
8. \(a(2a + b)\)
9. \(-5(3x^2 - 7x + 9)\)

Application

10. **TENNIS** The perimeter of a tennis court is 228 feet. The length of the court is 6 feet more than twice the width. What are the dimensions of the tennis court?

Practice and Apply

Find each product.
11. \(7(2n + 5)\)
12. \((1 + 4b)6\)
13. \(t(t - 9)\)
14. \((x + 5)x\)
15. \(-a(7a + 6)\)
16. \(y(3 + 2y)\)
17. \(4n(10 + 2n)\)
18. \(-3x(6x - 4)\)
19. \(3y(y^2 - 2)\)
20. \(ab(a^2 + 7)\)
21. \(5x(x + y)\)
22. \(4m(m^2 - m)\)
23. \(7(-2x^2 + 5x - 11)\)
24. \(-3y(6 - 9y + 4y^2)\)
25. \(4c(c^3 + 7c - 10)\)
26. \(6x^2(-2x^3 + 8x + 1)\)

Solve each equation.
27. \(30 = 6(-2w + 3)\)
28. \(-3(2a - 12) = 3a - 45\)

29. **BASKETBALL** The dimensions of high school basketball courts are different than the dimensions of college basketball courts, as shown in the table. Use the information in the table to find the length and width of each court.

30. **BOXES** A box large enough to hold 43,000 liters of water was made from one large sheet of cardboard.
   a. Write a polynomial that represents the area of the cardboard used to make the box. Assume the top and bottom of the box are the same. (Hint: \(2x^2 + 2y^2\) \((6x - 2y) = 12x^2 + 8xy - 4y^2\))
   b. If \(x\) is 1.2 meters and \(y\) is 0.1 meter, what is the total amount of cardboard in square meters used to make the box?

31. **CRITICAL THINKING** You have seen how algebra tiles can be used to connect multiplying a polynomial by a monomial and the Distributive Property. Draw a model and write a sentence to show how to multiply two binomials: \((a + b)(c + d)\).
32. **WRITING IN MATH**  Answer the question that was posed at the beginning of the lesson.

How is the Distributive Property used to multiply a polynomial by a monomial?
Include the following in your answer:
• a description of the Distributive Property, and
• an example showing the steps used to multiply a polynomial and a monomial.

33. What is the product of $2x$ and $x - 8$?

- **A** $2x - 8$
- **B** $2x^2 - 8$
- **C** $2x^2 - 16$
- **D** $2x^2 - 16x$

34. The area of the rectangle is 252 square centimeters. Find the length of the longer side.

- **A** 18 cm
- **B** 16 cm
- **C** 14 cm
- **D** 10 cm

---

**Maintain Your Skills**

**Mixed Review**  Find each sum or difference. *(Lessons 13-2 and 13-3)*

35. $(2x - 1) + 5x$  
36. $(9a + 3a^2) + (a + 4)$

37. $(y^2 + 6y + 2) + (3y^2 - 8y + 12)$  
38. $(4x - 7) - (2x + 2)$

39. $(9x + 8y) - (x - 3y)$  
40. $(13n^2 + 6n + 5) - (6n^2 + 5)$

41. **STATISTICS**  Describe two ways that a graph of sales of several brands of cereal could be misleading. *(Lesson 12-5)*

State whether each transformation of the triangles is a **reflection**,** translation**, or **rotation**. *(Lesson 10-3)*

42. 

43. 

---

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL**  Complete each table to find the coordinates of four points through which the graph of each function passes.
*(To review using tables to find ordered pair solutions, see Lesson 8-2.)*

44. $y = 4x$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$4x$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

45. $y = 2x^2 - 3$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$2x^2 - 3$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

46. $y = x^3 + 1$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$x^3 + 1$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Determine whether a function is linear or nonlinear.

**Nonlinear Functions**
In Lesson 8-2, you learned that linear functions have graphs that are straight lines. These graphs represent constant rates of change. Nonlinear functions do not have constant rates of change. Therefore, their graphs are not straight lines.

**Example 1** Identify Functions Using Graphs
Determine whether each graph represents a linear or nonlinear function. Explain.

**Example 2** Identify Functions Using Equations
Determine whether each equation represents a linear or nonlinear function.

- **a.** \( y = 10x \)
  - This is linear because it can be written as \( y = 10x + 0 \).

- **b.** \( y = \frac{3}{x} \)
  - This is nonlinear because \( x \) is in the denominator and the equation cannot be written in the form \( y = mx + b \).
The tables represent the functions in Example 2. Compare the rates of change.

### Linear

<table>
<thead>
<tr>
<th>x</th>
<th>y = 10x</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
</tr>
</tbody>
</table>

The rate of change is constant.

### Nonlinear

<table>
<thead>
<tr>
<th>x</th>
<th>y = (\frac{3}{x})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0.75</td>
</tr>
</tbody>
</table>

The rate of change is not constant.

A nonlinear function does not increase or decrease at the same rate. You can check this by using a table.

**Example 3** Identify Functions Using Tables

Determine whether each table represents a linear or nonlinear function.

**a.**

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>120</td>
</tr>
<tr>
<td>15</td>
<td>100</td>
</tr>
<tr>
<td>20</td>
<td>80</td>
</tr>
<tr>
<td>25</td>
<td>60</td>
</tr>
</tbody>
</table>

As \(x\) increases by 5, \(y\) decreases by 20. So this is a linear function.

**b.**

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>8</td>
<td>64</td>
</tr>
</tbody>
</table>

As \(x\) increases by 2, \(y\) increases by a greater amount each time. So this is a nonlinear function.

Some nonlinear functions are given special names.

**Key Concept** Quadratic and Cubic Functions

A **quadratic function** is a function that can be described by an equation of the form \(y = ax^2 + bx + c\), where \(a \neq 0\).

A **cubic function** is a function that can be described by an equation of the form \(y = ax^3 + bx^2 + cx + d\), where \(a \neq 0\).

Examples of these and other nonlinear functions are shown below.
Lesson 13-5
Linear and Nonlinear Functions

1. Describe two methods for determining whether a function is linear.

2. Explain whether a company would prefer profits that showed linear growth or exponential growth.

3. OPEN ENDED Use newspapers, magazines, or the Internet to find real-life examples of nonlinear situations.

Determine whether each graph, equation, or table represents a linear or nonlinear function. Explain.

4.  

5.  

6. \(y = \frac{x}{5}\)  

7. \(xy = 12\)

8. \[
\begin{array}{c|c}
 x & y \\
-4 & 13 \\
-2 & 0 \\
0 & 4 \\
2 & 0 \\
\end{array}
\]

9. \[
\begin{array}{c|c}
 x & y \\
8 & 19 \\
9 & 22 \\
10 & 25 \\
11 & 28 \\
\end{array}
\]

10. Which rule describes a nonlinear function?

- A \(x + y = 100\)
- B \(y = \frac{8}{x}\)
- C \(9 = 11x - y\)
- D \(x = y\)

**Example 4: Describe a Linear Function**

**Multiple-Choice Test Item**

Which rule describes a linear function?

- A \(y = 7x^3 + 2\)
- B \(y = (x - 1)5x\)
- C \(4x + 3y = 12\)
- D \(-2x^2 + 6y = 8\)

**Read the Test Item**

A rule describes a relationship between variables. A rule that can be written in the form \(y = mx + b\) describes a relationship that is linear.

**Solve the Test Item**

- \(y = 7x^3 + 2\) → cubic equation
- \(-2x^2 + 6y = 8\) → quadratic equation

You can eliminate choices A and D.

- \(y = (x - 1)5x\)
- \(y = 5x^2 - 5x\)

This is a quadratic equation. Eliminate choice B.

**CHECK**

\(4x + 3y = 12\) → \(y = -\frac{4}{3}x + 4\)

This equation is in the form \(y = mx + b\). ✓

**Test-Taking Tip**

Find out if there is a penalty for incorrect answers. If there is no penalty, making an educated guess can only increase your score, or at worst, leave your score the same.
The trend in farm income can be modeled with a nonlinear function. Visit www.pre-alg.com/webquest to continue work on your WebQuest project.

Determine whether each graph, equation, or table represents a linear or nonlinear function. Explain.

11. [Graph]
12. [Graph]
13. [Graph]
14. [Graph]
15. [Graph]
16. [Graph]
17. \( y = 0.9x \)
18. \( y = x^3 + 2 \)
19. \( y = \frac{3x}{4} \)
20. \( 2x + 3y = 12 \)
21. \( y = 4^x \)
22. \( xy = -6 \)
23. |\( x \) |\( y \) |
<table>
<thead>
<tr>
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<tbody>
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<td>11</td>
<td>-8</td>
</tr>
<tr>
<td>13</td>
<td>-14</td>
</tr>
<tr>
<td>15</td>
<td>-20</td>
</tr>
</tbody>
</table>
24. |\( x \) |\( y \) |
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<td>9</td>
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<td>16</td>
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25. |\( x \) |\( y \) |
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<td>4</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
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26. |\( x \) |\( y \) |
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</thead>
<tbody>
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<td>20</td>
</tr>
<tr>
<td>-9</td>
<td>18</td>
</tr>
<tr>
<td>-8</td>
<td>16</td>
</tr>
<tr>
<td>-7</td>
<td>14</td>
</tr>
</tbody>
</table>

27. TECHNOLOGY The graph shows the increase of trademark applications for internet-related products or services. Would you describe this growth as linear or nonlinear? Explain.

Online Research Data Update Is the growth of the Internet itself linear or nonlinear? Visit www.pre-alg.com/data_update to learn more.

28. CRITICAL THINKING Are all graphs of straight lines linear functions? Explain.

USA TODAY Snapshots®

Web leads trademark surge
Led by new online businesses, trademark applications jumped 32% last year to 265,342. Applications for Internet-related products or services:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>TM</td>
<td>3,059</td>
<td>8,212</td>
<td>9,406</td>
<td>12,235</td>
<td>33,731</td>
<td></td>
</tr>
</tbody>
</table>

By Anne R. Carey and Nancy E. Mullins, USA TODAY
29. **PATENTS**  The table shows the years in which the first six million patents were issued. Is the number of patents issued a linear function of time? Explain.

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of Patents Issued</th>
</tr>
</thead>
<tbody>
<tr>
<td>1911</td>
<td>1 million</td>
</tr>
<tr>
<td>1936</td>
<td>2 million</td>
</tr>
<tr>
<td>1961</td>
<td>3 million</td>
</tr>
<tr>
<td>1976</td>
<td>4 million</td>
</tr>
<tr>
<td>1991</td>
<td>5 million</td>
</tr>
<tr>
<td>1999</td>
<td>6 million</td>
</tr>
</tbody>
</table>

Source: New York Times

30. **WRITING IN MATH**  Answer the question that was posed at the beginning of the lesson.

**How can you determine whether a function is linear?**

Include the following in your answer:

- a list of ways in which a function can be represented, and
- an explanation of how each representation can be used to identify the function as linear or nonlinear.

31. Which equation represents a linear function?

- A) \( y = \frac{1}{2}x \)
- B) \( 3xy = 12 \)
- C) \( x^2 - 1 = y \)
- D) \( y = x(x + 4) \)

32. Determine which general rule represents a nonlinear function if \( a > 1 \).

- A) \( y = ax \)
- B) \( y = \frac{x}{a} \)
- C) \( y = a^x \)
- D) \( y = a + x \)

## Maintain Your Skills

**Mixed Review**

Find each product.  (Lesson 13-4)

33. \( t(4 + 9t) \)

34. \( 5n(-1 + 3n) \)

35. \( (a - 2b)ab \)

Find each difference.  (Lesson 13-3)

36. \( (2x + 7) - (x - 1) \)

37. \( (4x + y) - (5x + y) \)

38. \( (6a - a^2) - (8a + 3) \)

39. **GEOMETRY**  Classify a \(65^\circ\) angle as acute, obtuse, right, or straight.  (Lesson 9-3)

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL**  Use a table to graph each line.

(To review graphing equations, see Lesson 8-3.)

40. \( y = -x \)

41. \( y = x - 4 \)

42. \( y = 2x + 2 \)

43. \( y = -\frac{1}{2}x + 3 \)

**Practice Quiz 2**  Lessons 13-4 and 13-5

Find each product.  (Lesson 13-4)

1. \( c(2c^2 - 8) \)

2. \( (4x + 2)3x \)

3. \( a^2(5 + a + 2a^2) \)

Determine whether each equation represents a linear or nonlinear function.

Explain.  (Lesson 13-5)

4. \( y = 9x \)

5. \( y = 0.25x^3 \)
Graphing Quadratic and Cubic Functions

What You’ll Learn

- Graph quadratic functions.
- Graph cubic functions.

How are functions, formulas, tables, and graphs related?

You can find the area of a square \( A \) by squaring the length of a side \( s \). This relationship can be represented in different ways.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Table</th>
</tr>
</thead>
</table>
| Area equals length of a side squared. \[ A = s^2 \] | \[ \begin{array}{ccc}
 s & s^2 & (s, A) \\
 0 & 0^2 = 0 & (0, 0) \\
 1 & 1^2 = 1 & (1, 1) \\
 2 & 2^2 = 4 & (2, 4) \\
\end{array} \] |

a. The volume of cube \( V \) equals the cube of the length of an edge \( a \). Write a formula to represent the volume of a cube as a function of edge length.
b. Graph the volume as a function of edge length. (Hint: Use values of \( a \) like 0, 0.5, 1, 1.5, 2, and so on.)

**Example 1** Graph Quadratic Functions

Graph each function.
a. \( y = 2x^2 \)

Make a table of values, plot the ordered pairs, and connect the points with a curve.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 2x^2 )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.5</td>
<td>2(-1.5)^2 = 4.5</td>
<td>(-1.5, 4.5)</td>
</tr>
<tr>
<td>-1</td>
<td>2(-1)^2 = 2</td>
<td>(-1, 2)</td>
</tr>
<tr>
<td>0</td>
<td>2(0)^2 = 0</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>1</td>
<td>2(1)^2 = 2</td>
<td>(1, 2)</td>
</tr>
<tr>
<td>1.5</td>
<td>2(1.5)^2 = 4.5</td>
<td>(1.5, 4.5)</td>
</tr>
</tbody>
</table>

**QUADRATIC FUNCTIONS** In Lesson 13-5, you saw that functions can be represented using graphs, equations, and tables. This allows you to graph quadratic functions such as \( A = s^2 \) using an equation or a table of values.
You can also write a rule from a verbal description of a function, and then graph.

**Example 2 Use a Function to Solve a Problem**

**SKYDIVING** The distance in feet that a skydiver falls is equal to sixteen times the time squared, with the time given in seconds. Graph this function and estimate how far he will fall in 4.5 seconds.

**Words** Distance is equal to sixteen times the time squared.

**Variables** Let \( d \) = the distance in feet and \( t \) = the time in seconds.

**Equation**

\[
\begin{align*}
\text{Distance} & \quad \text{is equal to} \quad \text{sixteen} \quad \text{times} \quad \text{the time} \quad \text{squared.} \\
\end{align*}
\]

The equation is \( d = 16t^2 \). Since the variable \( t \) has an exponent of 2, this function is nonlinear. Now graph \( d = 16t^2 \). Since time cannot be negative, use only positive values of \( t \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( d = 16t^2 )</th>
<th>((t, d))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>16(0)^2 = 0</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>1</td>
<td>16(1)^2 = 16</td>
<td>(1, 16)</td>
</tr>
<tr>
<td>2</td>
<td>16(2)^2 = 64</td>
<td>(2, 64)</td>
</tr>
<tr>
<td>3</td>
<td>16(3)^2 = 144</td>
<td>(3, 144)</td>
</tr>
<tr>
<td>4</td>
<td>16(4)^2 = 256</td>
<td>(4, 256)</td>
</tr>
<tr>
<td>5</td>
<td>16(5)^2 = 400</td>
<td>(5, 400)</td>
</tr>
<tr>
<td>6</td>
<td>16(6)^2 = 576</td>
<td>(6, 576)</td>
</tr>
</tbody>
</table>

By looking at the graph, we find that in 4.5 seconds, the skydiver will fall approximately 320 feet. You could find the exact distance by substituting 4.5 for \( t \) in the equation \( d = 16t^2 \).
CUBIC FUNCTIONS You can also graph cubic functions such as the formula for the volume of a cube by making a table of values.

Example 3 Graph Cubic Functions

Graph each function.

a. \( y = x^3 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = x^3 )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.5</td>
<td>(-1.5)(^3) = -3.4</td>
<td>(-1.5, -3.4)</td>
</tr>
<tr>
<td>-1</td>
<td>(-1)(^3) = -1</td>
<td>(-1, -1)</td>
</tr>
<tr>
<td>0</td>
<td>(0)(^3) = 0</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>1</td>
<td>(1)(^3) = 1</td>
<td>(1, 1)</td>
</tr>
<tr>
<td>1.5</td>
<td>(1.5)(^3) ≈ 3.4</td>
<td>(1.5, 3.4)</td>
</tr>
</tbody>
</table>

b. \( y = x^3 - 1 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = x^3 - 1 )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.5</td>
<td>(-1.5)(^3) - 1 \approx -4.4</td>
<td>(-1.5, -4.4)</td>
</tr>
<tr>
<td>-1</td>
<td>(-1)(^3) - 1 = -2</td>
<td>(-1, -2)</td>
</tr>
<tr>
<td>0</td>
<td>(0)(^3) - 1 = -1</td>
<td>(0, -1)</td>
</tr>
<tr>
<td>1</td>
<td>(1)(^3) - 1 = 0</td>
<td>(1, 0)</td>
</tr>
<tr>
<td>1.5</td>
<td>(1.5)(^3) - 1 \approx 2.4</td>
<td>(1.5, 2.4)</td>
</tr>
</tbody>
</table>

Check for Understanding

Concept Check

1. Describe one difference between the graph of \( y = nx^2 \) and the graph of \( y = nx^3 \) for any rational number \( n \).
2. Explain how to determine whether a function is quadratic.
3. OPEN ENDED Write a quadratic function and explain how to graph it.

Guided Practice

Graph each function.

4. \( y = x^2 \)
5. \( y = -2x^2 \)
6. \( y = x^2 + 1 \)
7. \( y = -x^3 \)
8. \( y = 0.5x^3 \)
9. \( y = x^3 - 2 \)

Application

10. GEOMETRY A cube has edges measuring \( a \) units.
    a. Write a quadratic equation for the surface area \( S \) of the cube.
    b. Graph the surface area as a function of \( a \). (Hint: Use values of \( a \) like 0, 0.5, 1, 1.5, 2, and so on.)

Practice and Apply

Graph each function.

11. \( y = 3x^2 \)
12. \( y = 0.5x^2 \)
13. \( y = -x^2 \)
14. \( y = 3x^3 \)
15. \( y = -2x^3 \)
16. \( y = -0.5x^2 \)
17. \( y = 2x^3 \)
18. \( y = 0.1x^3 \)
19. \( y = x^3 + 1 \)
20. \( y = x^2 - 3 \)
21. \( y = \frac{1}{2}x^2 + 1 \)
22. \( y = \frac{1}{3}x^3 + 2 \)

Homework Help

For Exercises See Examples
11–22 1, 3
31–33 2

Extra Practice See page 757.
23. Graph \( y = x^2 - 4 \) and \( y = -4x^3 \). Are these equations functions? Explain.

24. Graph \( y = x^2 \) and \( y = x^3 \) in the first quadrant on the same coordinate plane. Explain which graph shows faster growth.

The **maximum point** of a graph is the point with the greatest \( y \) value coordinate. The **minimum point** is the point with the least \( y \) value coordinate. Find the coordinates of each point.

- **25.** the maximum point of the graph of \( y = -x^2 + 7 \)
- **26.** the minimum point of the graph of \( y = x^2 - 6 \)

Graph each pair of equations on the same coordinate plane. Describe their similarities and differences.

- **27.** \( y = x^2 \) and \( y = 3x^2 \)
- **28.** \( y = 0.5x^3 \) and \( y = 2x^3 \)
- **29.** \( y = 2x^2 \) and \( y = -2x^2 \)
- **30.** \( y = x^3 \) and \( y = x^3 - 3 \)

**CONSTRUCTION** For Exercises 31–33, use the information below and the figure at the right.

A dog trainer is building a dog pen with a 100-foot roll of chain link fence.

- **31.** Write an equation to represent the area \( A \) of the pen.
- **32.** Graph the equation you wrote in Exercise 31.
- **33.** What should the dimensions of the dog pen be to enclose the maximum area inside the fence? (*Hint:* Find the coordinates of the maximum point of the graph.)

**GEOMETRY** Write a function for each of the following. Then graph the function in the first quadrant.

- **34.** the volume \( V \) of a rectangular prism as a function of a fixed height of 2 units and a square base of varying lengths \( s \)
- **35.** the volume \( V \) of a cylinder as a function of a fixed height of 0.2 unit and radius \( r \)

**CRITICAL THINKING** Describe how you can find real number solutions of the quadratic equation \( ax^2 + bx + c = 0 \) from the graph of the quadratic function \( y = ax^2 + bx + c \).

**37.** **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**How are functions, formulas, tables, and graphs related?**

Include the following in your answer:
- an explanation of how to make a graph by using a rule, and
- an explanation of how to write a rule by using a graph.

**38.** Which equation represents the graph at the right?

- **A** \( y = 4x^2 \)
- **B** \( y = -4x^2 \)
- **C** \( y = 4x^3 \)
- **D** \( y = -4x^3 \)
39. For a certain frozen pizza, as the cost goes from $2 to $4, the demand can be modeled by the formula $y = -10x^2 + 60x + 180$, where $x$ represents the cost and $y$ represents the number of pizzas sold. Estimate the cost that will result in the greatest demand.

\[ \text{A} \quad \$0 \quad \text{B} \quad \$2 \quad \text{C} \quad \$3 \quad \text{D} \quad \$8 \]

**Maintain Your Skills**

**Mixed Review**

42. **SCIENCE** The graph shows how water vapor pressure increases as the temperature increases. Is this relationship linear or nonlinear? Explain. *(Lesson 13-5)*

Find each product. *(Lesson 13-4)*

43. $(2x - 4)5$  
44. $n(n + 6)$  
45. $3y(8 - 7y)$

Write an equation in slope-intercept form for the line passing through each pair of points. *(Lesson 8-7)*

46. $(3, 6)$ and $(0, 9)$  
47. $(2, 5)$ and $(-1, -7)$  
48. $(-4, -3)$ and $(8, 6)$

**WebQuest Internet Project**

**Family Farms**

It is time to complete your project. Use the information and data you have gathered to prepare a Web page about farming or ranching in the United States. Be sure to include at least five graphs or tables that show statistics about farming or ranching and at least one scatter plot that shows a farming or ranching statistic over time, from which you can make predictions.

[www.pre-alg.com/webquest](http://www.pre-alg.com/webquest)
Families of Quadratic Functions

A quadratic function can be described by an equation of the form $ax^2 + bx + c$, where $a \neq 0$. The graph of a quadratic function is called a parabola. Recall that families of linear graphs share the same slope or $y$-intercept. Similarly, families of parabolas share the same maximum or minimum point, or have the same shape.

Graph $y = x^2$ and $y = x^2 + 4$ on the same screen and describe how they are related.

**Step 1** Enter the function $y = x^2$.
- Enter $y = x^2$ as Y1.
  KEYS: $y \leftarrow x^2 \rightarrow \text{ENTER}$

**Step 2** Enter the function $y = x^2 + 4$.
- Enter $y = x^2 + 4$ as Y2.
  KEYS: $y \leftarrow x^2 + 4 \rightarrow \text{ENTER}$

**Step 3** Graph both quadratic functions on the same screen.
- Display the graph.
  KEYS: ZOOM 6

The first function graphed is Y1 or $y = x^2$. The second is Y2 or $y = x^2 + 4$. Press TRACE and move along each function by using the right and left arrow keys. Move from one function to another by using the up and down arrow keys.

The graphs are similar in that they are both parabolas. However, the graph of $y = x^2$ has its vertex at $(0, 0)$, whereas the graph of $y = x^2 + 4$ has its vertex at $(0, 4)$.

**Exercises**

1. Graph $y = x^2$, $y = x^2 - 5$, and $y = x^2 - 3$ on the same screen and draw the parabolas on grid paper. Compare and contrast the three parabolas.

2. Make a conjecture about how adding or subtracting a constant $c$ affects the graph of a quadratic function.

3. The three parabolas at the right are graphed in the standard viewing window and have the same shape as the graph of $y = x^2$. Write an equation for each, beginning with the lowest parabola.

4. Clear all functions from the Y= menu. Enter $y = 0.4x^2$ as Y1, $y = x^2$ as Y2, and $y = 3x^2$ as Y3. Graph the functions in the standard viewing window on the same screen. Then draw the graphs on the same coordinate grid. How does the shape of the parabola change as the coefficient of $x^2$ increases?

www.pre-alg.com/other_calculator_keystrokes
Polynomials

Concept Summary

- A polynomial is an algebraic expression that contains one or more monomials.
- A binomial has two terms and a trinomial has three terms.
- The degree of a monomial is the sum of the exponents of its variables.

State whether \(x^3 - 2xy\) is a monomial, binomial, or trinomial. Then find the degree.

The expression is the difference of two monomials. So it is a binomial.

\(x^3\) has degree 3, and \(-2xy\) has degree 1 + 1 or 2. So, the degree of \(x^3 - 2xy\) is 3.

Exercises

Determine whether each expression is a polynomial. If it is, classify it as a monomial, binomial, or trinomial. See Example 1 on page 669.

9. \(c^2 + 3\)
10. \(-5\)
11. \(4t^4\)
12. \(\frac{6}{a} + b\)
13. \(3x^2 + 4x - 2\)
14. \(x + y\)
15. \(\sqrt{n}\)
16. \(1 + 3x + 5x^2\)

Find the degree of each polynomial. See Examples 2 and 3 on page 670.

17. \(2x\)
18. \(5xy\)
19. \(3a^2b\)
20. \(n^2 - 4\)
21. \(x^6 + y^6\)
22. \(2xy + 6yz^2\)
23. \(2x^5 + 9x + 1\)
24. \(x^2 + xy^2 - y^4\)
Adding Polynomials

Concept Summary
- To add polynomials, add like terms.

Example
Find \((5x^2 - 8x + 2) + (x^2 + 6x)\).

\[
\begin{align*}
5x^2 & - 8x + 2 \\
(+) & x^2 + 6x \\
\hline \\
6x^2 & - 2x + 2 \\
\end{align*}
\]

Align like terms.

Add.

The sum is \(6x^2 - 2x + 2\).

Exercises
Find each sum. See Example 1 on pages 674 and 675.
25. \(3b + 8\) 26. \(2x^2 + 3x - 4\) 27. \(4y^2 + 2y + 3\)
\((+5b - 5)\) \((+6x^2 - x + 5)\) \((+y^2 - 7)\)
28. \(9m - 3n + (10m + 4n)\) 29. \((-3y^2 + 2) + (4y^2 - y - 3)\)

Subtracting Polynomials

Concept Summary
- To subtract polynomials, subtract like terms or add the additive inverse.

Example
Find \((4x^2 + 7x + 4) - (x^2 + 2x + 1)\).

\[
\begin{align*}
4x^2 & + 7x + 4 \\
(-) & x^2 + 2x + 1 \\
\hline \\
3x^2 & + 5x + 3 \\
\end{align*}
\]

Align like terms.

Subtract.

The difference is \(3x^2 + 5x + 3\).

Exercises
Find each difference. See Examples 1 and 2 on pages 678 and 679.
30. \(a^2 + 15\) 31. \(4x^2 - 2x + 3\) 32. \(18y^2 + 3y - 1\)
\((-)3a^2 - 10\) \((-) x^2 + 2x - 4\) \((-) 2y^2 + \) \(+ 6\)
33. \((x + 8) - (2x + 7)\) 34. \((3n^2 + 7) - (n^2 - n + 4)\)

Multiplying a Polynomial by a Monomial

Concept Summary
- To multiply a polynomial and a monomial, use the Distributive Property.

Example
Find \(-3x(x + 8y)\).

\[
-3x(x + 8y) = -3x(x) + (-3x)(8y) \quad \text{Distributive Property}
\]

\[
= -3x^2 - 24xy \quad \text{Simplify.}
\]

Exercises
Find each product. See Example 1 on page 683.
35. \(5(4t - 2)\) 36. \((2x + 3y)7\) 37. \(k(6k + 3)\)
38. \(4d(2d - 5)\) 39. \(-2a(9 - a^2)\) 40. \(6(2x^2 + xy + 3y^2)\)
13-5 Linear and Nonlinear Functions

Concept Summary

Nonlinear functions do not have constant rates of change.

Example

Determine whether each graph, equation, or table represents a linear or nonlinear function. Explain.

a. Nonlinear; graph is not a straight line.

b. Linear; equation can be written as $y = mx + b$.

c. Linear; rate of change is constant.

Exercises Determine whether each graph, equation, or table represents a linear or nonlinear function. Explain. See Examples 1–3 on pages 687 and 688.

41. 

42. $y = \frac{x}{2}$

43. 

13-6 Graphing Quadratic and Cubic Functions

Concept Summary

Quadratic and cubic functions can be graphed by plotting points.

Example

Graph $y = -x^2 + 3$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = -x^2 + 3$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>$-(-2)^2 + 3$ = -1</td>
<td>(-2, -1)</td>
</tr>
<tr>
<td>-1</td>
<td>$-(-1)^2 + 3$ = 2</td>
<td>(-1, 2)</td>
</tr>
<tr>
<td>0</td>
<td>$-(0)^2 + 3$ = 3</td>
<td>(0, 3)</td>
</tr>
<tr>
<td>1</td>
<td>$-(1)^2 + 3$ = 2</td>
<td>(1, 2)</td>
</tr>
<tr>
<td>2</td>
<td>$-(2)^2 + 3$ = -1</td>
<td>(2, -1)</td>
</tr>
</tbody>
</table>

Exercises Graph each function. See Examples 1 and 3 on pages 692–694.

44. $y = x^2 + 2$

45. $y = -3x^2$

46. $y = x^3 - 2$

47. $y = x^3 + 1$

48. $y = -x^3$

49. $y = 2x^2 + 4$
Vocabulary and Concepts

1. Define polynomial.
2. Explain how the degree of a monomial is found.
3. OPEN ENDED  Draw the graph of a linear and a nonlinear function.

Skills and Applications

Determine whether each expression is a polynomial. If it is, classify it as a monomial, binomial, or trinomial.

4. $3x^3 - 2x + 7$
5. $6 + \frac{5}{m}$
6. $\frac{3}{5}p^4$

Find the degree of each polynomial.

7. $5ab^3$
8. $w^5 - 3w^3y^4 + 1$

Find each sum or difference.

9. $(5y + 8) + (-2y + 3)$
10. $(5a - 2b) + (-4a + 5b)$
11. $(-3m^3 + 5m - 9) + (7m^3 - 2m^2 + 4)$
12. $(6p + 5) - (3p - 8)$
13. $(5w - 3x) - (6w + 4x)$
14. $(-2s^2 + 4s - 7) - (6s^2 - 7s - 9)$

Find each product.

15. $x(3x - 5)$
16. $-5a(a^2 - b^2)$
17. $6p(-2p^2 + 3p - 4)$

Determine whether each graph, equation, or table represents a linear or nonlinear function. Explain.

18. 
19. $5x - 6y = 2$
20. 

Graph each function.

21. $y = 2x^2$
22. $y = \frac{1}{2}x^3$
23. $y = -x^2 + 3$

24. GEOMETRY  Refer to the rectangle.
   a. Write an expression for the perimeter of the rectangle.
   b. Find the value of $x$ if the perimeter is 14 inches.

25. STANDARDIZED TEST PRACTICE  The length of a garden is equal to 5 less than four times its width. The perimeter of the garden is 40 feet. Find the length of the garden.
   A. 1 ft  B. 5 ft  C. 10 ft  D. 15 ft
1. If \( n \) represents a positive number, which of these expressions is equivalent to \( \frac{n}{100} + \frac{n}{100} \)? (Lesson 1-3)
   - A) \( n^3 \)
   - B) \( 3n \)
   - C) \( n + 3 \)
   - D) \( 3(n + 1) \)

2. Connor sold 4 fewer tickets to the band concert than Miguel sold. Kylie sold 3 times as many tickets as Connor. If the number of tickets Miguel sold is represented by \( m \), which of these expressions represents the number of tickets that Kylie sold? (Lesson 3-6)
   - A) \( m - 4 \)
   - B) \( 4 - 3m \)
   - C) \( 3m - 4 \)
   - D) \( 3(m - 4) \)

3. Melissa’s family calculated that they drove an average of 400 miles per day during their three-day trip. They drove 460 miles on the first day and 360 miles on the second day. How many miles did they drive on the third day? (Lesson 5-7)
   - A) 340
   - B) 380
   - C) 410
   - D) 420

4. What is the ratio of the length of a side of a square to its perimeter? (Lesson 6-1)
   - A) \( \frac{1}{16} \)
   - B) \( \frac{1}{4} \)
   - C) \( \frac{1}{3} \)
   - D) \( \frac{1}{2} \)

5. The table shows values of \( x \) and \( y \), where \( x \) is proportional to \( y \). What are the missing values, \( S \) and \( T \)? (Lesson 6-3)
   - A) \( S = 36 \) and \( T = 3 \)
   - B) \( S = 21 \) and \( T = 15 \)
   - C) \( S = 15 \) and \( T = 21 \)
   - D) \( S = 3 \) and \( T = 36 \)

6. In the figure at the right, lines \( \ell \) and \( m \) are parallel. Choose two angles whose measures have a sum of 180°. (Lesson 10-2)
   - A) \( \angle 1 \) and \( \angle 5 \)
   - B) \( \angle 2 \) and \( \angle 8 \)
   - C) \( \angle 2 \) and \( \angle 5 \)
   - D) \( \angle 4 \) and \( \angle 8 \)

7. The point represented by coordinates \((4, -6)\) is reflected across the \( x \)-axis. What are the coordinates of the image? (Lesson 10-3)
   - A) \((-6, 4)\)
   - B) \((-4, -6)\)
   - C) \((-4, 6)\)
   - D) \((4, 6)\)

8. If \( 2x^2 - 3x + 7 \) is subtracted from \( 4x^2 + 6x - 3 \), what is the difference? (Lesson 13-3)
   - A) \( 2x^2 + 9x - 10 \)
   - B) \( 2x^2 + 3x + 4 \)
   - C) \( -2x^2 + 3x + 4 \)
   - D) \( -2x^2 - 9x + 10 \)

9. Which function includes all of the ordered pairs in the table? It may help you to sketch a graph of the points. (Lesson 13-6)
   - A) \( y = -x^2 \)
   - B) \( y = -2x \)
   - C) \( y = -x + 2 \)
   - D) \( y = x^2 \)
10. The table shows the number of sandwiches sold during twenty lunchtimes. What is the mode? (Lesson 5-8)

<table>
<thead>
<tr>
<th>Number of Sandwiches Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
</tr>
<tr>
<td>16</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>12</td>
</tr>
</tbody>
</table>

11. One machine makes plastic containers at a rate of 360 containers per hour. A newer machine makes the same containers at a rate of 10 containers per minute. If both machines are run for four hours, how many containers will they make? (Lesson 6-1)

12. What percent of 275 is 165? (Lesson 6-5)

13. Mrs. Rosales can spend $8200 on equipment for the computer lab. Each computer costs $850 and each printer costs $325. Mrs. Rosales buys 8 computers. Write an inequality that can be used to find \( p \), the number of printers she could buy. (Lesson 7-6)

14. What is the \( y \)-intercept of the graph shown at the right? Each square represents 1 unit. (Lesson 8-6)

15. The area of a triangle is \( \frac{1}{2}(b \times h) \). What is the area of the kite? (Lesson 10-5)

16. Brooke wants to fill her new aquarium two-thirds full of water. The aquarium dimensions are 20 inches by 20 inches by \( 8\frac{1}{2} \) inches. What volume of water, in cubic inches, is needed? (Lesson 11-2)

17. Let \( s = 3x^2 - 2x - 1 \) and \( t = -2x^2 + x + 2 \). Find \( s + t \). (Lesson 13-2)

18. The perimeter of a soccer field is 1040 feet. The length of the field is 40 feet more than 2 times the width. What is the length of the field? (Lesson 13-4)

19. An artist created a sculpture using five cylindrical posts. Each post has a diameter of 12 inches. The heights of the posts are 6 feet, 5 feet, 4 feet, 3 feet, and 2 feet. (Lesson 11-2)

a. What is the total volume of all five posts? \( V = \pi r^2h \) is the formula for the volume of a cylinder. Use \( \pi = 3.14 \).

b. The posts are made of a material whose density is 12 pounds per cubic foot. How much does the sculpture weigh?

20. Refer to the table below. (Lesson 13-6)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>-3</td>
</tr>
<tr>
<td>0</td>
<td>-4</td>
</tr>
<tr>
<td>1</td>
<td>-3</td>
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<tr>
<td>2</td>
<td>0</td>
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